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MANAGING INVESTMENT RISKS OF INSTITUTIONAL PRIVATE EQUITY INVESTORS
- THE CHALLENGE OF ILLIQUIDITY -

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Managing Investment Risks of Institutional Private Equity Investors

- The Challenge of Illiquidity -

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Abstract

Since private equity investments are not publicly traded, a key issue in measuring investment risks of institutional private equity investors arises from a careful measurement of investment returns in the first place. Prices of private equity investments are typically observed at low frequency and are determined by transactions under low liquidity. This contribution highlights useful approaches to the problem of return measurement under conditions of illiquidity. Then, specific risk management issues, including asset allocation issues, are discussed.

JEL classification: G1, G2

Keywords: **private equity, risk/return measurement, net asset values, cash flows, illiquidity, stale pricing, risk management, asset allocation.**

1. Introduction

Private equity has become an increasingly important alternative asset class for institutional investors as it may offer return as well as diversification benefits relative to traditional stock and bond market investments. In fact, the market for private equity investments has grown dramatically over the 1998 to 2000 period. However, the economic downturn during 2001 to 2003 had a strong negative impact on the funds raised by the private equity industry. Nevertheless, it is common wisdom that private equity will again become an important source of corporate financing and, thereby, an important driver in economic prosperity.

From an economic point of view, one of the most important advantages of private equity compared to public equity is to overcome the free-rider problem in corporate control. While dispersed ownership as the typical ownership structure in public equity markets does not generate sufficient incentives to undertake costly control activities, private equity markets typically go along with concentrated ownership in portfolio companies. This is because the private equity investor normally holds a large part of equity in his portfolio company. For that reason he exercises a continuous monitoring activity. The private equity investor is typically by itself a fund where a given number of private or institutional investors, called limited partners, have paid in their capital. The fund is run by a management team called general partner. Of course, a conflict of interests between the general and the limited partners could arise. Normally, however, this problem will be avoided as the number of limited partners is not too high and the general partner has either invested his own money in the fund or is paid according to some purposeful incentive scheme. Whether these problems may become more serious in the case of a fund of funds construction, may be left open here. In such cases the outside investor has only a contractual relationship with the management team of the fund of funds; the allocation of capital to different private equity funds is made by the management team.

From an investors point of view it is important to note that several empirical results, which are available particularly for venture capital as a special segment of private equity focused on financing high risk start-up firms, indicate that this kind of alternative investment may indeed offer desirable risk-return and particularly diversification characteristics.[1]

Despite these potential benefits it is important to point out that the lack of an organised secondary market for alternative investments comes along with low liquidity or even illiquidity in the transfer of alternative asset ownership. Hence, a major drawback of the private equity asset class is its liquidity risk. The latter can manifest itself with the impossibility to transact at a given point in time and/or with the occurrence of substantial transaction cost.[2]

There are two major consequences of the lack of an organised secondary market. First, liquidity –jointly with investment risk and return– will play a major role in a fund manager's decision to include private equity in her managed fund of assets. Second, we argue here that liquidity has an additional indirect impact on the decision to invest in that it has an important impact on the measurement of returns of relatively illiquid assets. As risk is statistically derived from return observations, liquidity will also play a key role in an accurate measurement of private equity investment risk.

In the following section we discuss useful approaches to the problem of liquidity related return and risk measurement. Hereby, we present two important methods how return characteristics can be measured in the context of illiquid markets. The first method presented in Section 2.1 relies on reported asset values, while the second method presented in Section 2.2 is based on observable, although infrequent cash flows. Given that private equity returns were determined, Section 3 then discusses the consequences for risk management as well as asset allocation issues. Obviously, most of the risk management and asset allocation methods that apply to public equity will also apply to private equity. However, there are some issues, especially related to the problem of illiquidity and measurement biases, which are specific in risk management and asset allocation of private equity investments. These issues are discussed in Sections 3.1 and 3.2. Section 4 contains a brief conclusion and gives a topic outlook.

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[1] See e.g. Schilit (1993), Cochrane (2001), Chen et al. (2002), Emery (2003), and the literature given therein. For an overview of venture capital see also for example Gompers and Lerner (1999).

[2] Such transaction costs are for example given by high market impact costs, the cost of searching potential buyers and sellers as well as potential agency costs related to changes in the ownership structure given a desired transfer of assets.

2. Measuring Private Equity Returns and Risk

It has already been mentioned that a private equity investment can be undertaken directly or indirectly via a so called private equity fund. Therefore, risk-/return characteristics of private equity investments can basically be defined from two different perspectives. Either one is interested in assessing the return distribution of an investment in a single company seeking for equity financing or in assessing the return distribution of an investment in a private equity fund. As far as risk management issues are concerned the first perspective is especially relevant from the viewpoint of a general partner, as he is supposed to make congruent decisions with respect to the allocation of capital provided by limited partners to portfolio firms. The second perspective is relevant for a private or institutional investor considering to act as a limited partner, i.e. to invest money in a private equity fund.[3] Hence, when talking about return distributions one should make clear as to what kind of return processes he is referring to: returns generated at the level of a single portfolio firm, labelled as transaction level, or returns generated at the level of a private equity fund, labelled as the fund level.

As this article deals with risk management issues of institutional investors we are focussing on return distribution at the fund level. However, much of the methodological issues raised here could safely be applied to return distributions at the transaction level as well. Hence, these different perspectives are not that important for what follows here.

From an economic point of view, the most important characteristic of private equity investments are missing or highly imperfect secondary markets. As a consequence, for any single fund investment there are only a few points in time for which transaction prices can be observed: when limited partners pay in their capital and when the investment is liquidated. Usually, such transactions do not happen very frequently. Over a fund's lifetime, normally 5 to 10 years, one would observe not more than a handful of cash flow transactions between the fund and its limited partners. Moreover, even if cash flows would arise more frequently, the lack of reliable information with respect to the market value of a particular private equity fund will not be offset. As a consequence, no intermediate series of historical returns is available. Therefore, realized returns of private equity investments can only be observed by looking at the cash flow stream generated over a fund's lifetime. However, one should be careful in comparing cash flow based internal rates of return (*IRRs*) with return figures derived from the market

3 Obviously, this perspective is also relevant for a fund of funds manager.

value observation of a public equity investment. As an alternative, one could calculate stock based private equity returns on the basis of reported asset values, such as net asset values (*NAV*). However, these values do not represent market transactions and may be subject to rigidity due to smoothing activities undertaken by the fund management and/or subject to observational noise.

The problem of assessing return distributions of assets traded on markets subject to liquidity constraints has been first analyzed in the context of the asynchronous trading literature in finance. Scholes and Williams (1977), Roll (1981) and Cohen et al. (1983) consider the estimation of asset betas of relatively illiquid small capitalisation stocks. A more elaborated version of this approach has been presented by Lo and MacKinlay (1990). As they rely on the assumption that market prices of assets can be observed at least at some points in time, an extension of this approach to the issues in question here is not possible.[4] More recently and more appropriate, Getmansky et al. (2003) derive a related econometric time series model which considers return smoothing as a result of illiquidity in investment portfolios. This will be considered in more detail here.

Peng (2001) proposes an extension to repeat sales regression which was used in the real estate finance literature. The method is based on estimating time series returns of a portfolio of infrequently traded assets based on a cross-section of observed transaction prices for a subset of assets.

Other approaches to illiquidity include Longstaff (1995) who uses option-pricing theory in order to assess the maximum value of the ability to trade immediately in a liquid market. The model derives an upper bound for the value by assuming a trader with perfect foresight. In case the trader wants to sell, in a perfectly liquid market he may realize the maximum asset value governing at a given time period and, hence, realize the value of a lookback option. This option value then can be considered as an upper bound for the value of marketability to an investor with imperfect foresight.

While the Longstaff (1995) approach may be useful to derive upper bounds on discounts for private equity asset values, it does not relate to asset price dynamics and the measurement of returns and risk. As we do not assume to have a representative cross-section of assets, the approach by Peng (2001) seems to have too high data requirements. In the following we will therefore first focus on the approach of

4 For a more detailed description of this approach cf. Campbell et al. (1997, p. 84 n.).

Getmansky et al. (2003). Thereafter, we will discuss the implications of measuring cash flow based returns instead of asset value based returns.

2.1 Asset Value based Returns

In an ideal economy with frictionless and informational efficient markets, i.e. when transaction costs can be neglected and all information is immediately incorporated into market prices, classical finance models apply. In these models asset prices P_t fluctuate randomly and returns $R_t = P_t/P_{t-1} - 1$ are hence independent.[5]

In discrete time, i.e. for $t = 1, ..., T$, such ideal conditions can be formalised by assuming that the return R_t is independently and identically distributed (iid) with a common distribution function F. For our purposes we can think of R_t as the true period-t asset return of an investment in a private equity fund.

Under illiquidity, the true period-t asset return R_t will –by definition– be *unobservable*. Think, for instance, at a private equity investment in a single company. In this case, the value of this investment can be observed on the vintage date, i.e. the day the capital is injected into the company, and on the exit date, i.e. the day the private equity investor sells its stocks on the market. Frequently there are intermediate transactions, for instance, when there are additional financing rounds or when the private equity investors makes a partial sell-off from his stake. Hence, with respect to the value of a single portfolio company investment we have a handful of points in time where asset prices can be observed on the basis of transaction related prices. If, however, we think of a private equity investment, even this kind of restricted revelation of market values may break down. As the fund is typically invested in more than one company, cash flows between the fund and its investors can be regarded as pooled cash flows between the fund and all its portfolio companies. In this case the fund's asset values can be observed on the vintage date and on the liquidation date. However, as long as there are some intermediate cash flows between the fund and the investors putting these two values into relation does not reveal anything about the true return generated for the investors.

One way out of this problem is to check as to what extend intermediate true returns can be inferred from accounting based asset values. Specifically, private equity funds

[5] In some models it is assumed that not discrete but continuous returns are independent. For our purposes here, this difference is of minor importance. Actually, we assume discrete returns to be independent.

regularly disclose *NAV* on a semi-annual or even quarterly basis. These values are calculated along valuation guidelines developed in some kind of self-regulation context and are supposed to reflect the fair value of the whole investment portfolio. Moreover, under some circumstances the fair value should be derived within a marking-to-market framework. Nevertheless, it is quite obvious that *NAV* would only occasionally reflect the true market price, i.e. the price at which a fund's assets could be sold in an open market transaction. This may either be due to unavoidable valuation errors made by the general partner or due to a strategic disclosure policy followed by the latter.

However, it can be shown that at least under some circumstances it may be possible to derive true returns from observed *NAV*. Lets assume that we can, in fact, observe some intermediate proxy return Q_t. This, of course, implies a perceivable loss of information about the underlying distribution F of the true returns R_t. In particular, we loose information about the risk inherent in these returns. In order to gather at least some information on F one has to assume some model based relationship between the true returns R_t and the observed returns Q_t.

Figure 2.1 illustrates the given situation of a true price process P_t and two intermediate price estimates that deviate from the true prices. The deviating price estimates imply errors in the observed intermediate returns Q_t. The figure is set up as a simplifying situation for a private equity investment with initial investment P_0 at time zero and final liquidation value P_T at time T.

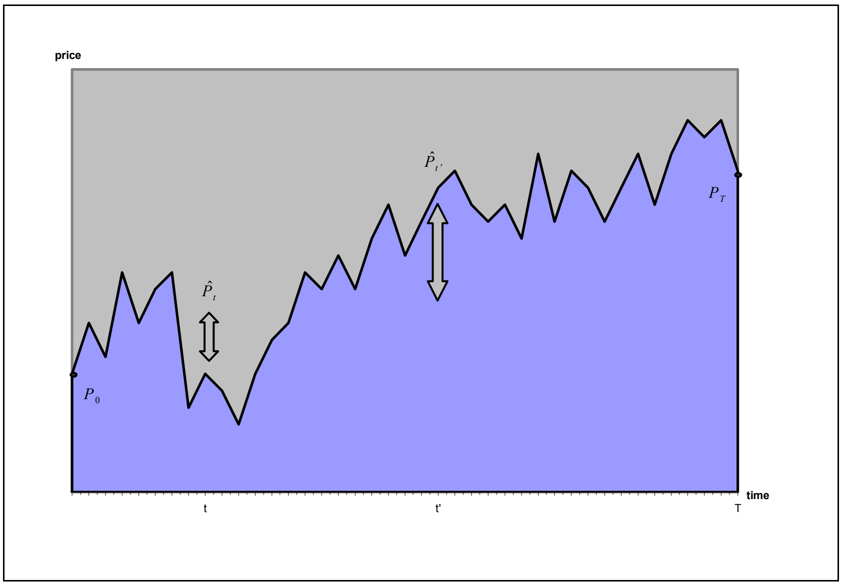

Fig. 2.1. True price process P_t and deviating price estimates at time t and t' each being reported before final liquidation time T.

With respect to modelling the relationship between true returns R_t and the observed returns Q_t's a huge number of different specifications may be conceivable. However, based on findings in the literature we believe that the following two different specifications for the time being are the most important ones.

2.1.2 Smoothed Proxy Observations

Getmansky et al. (2003) assume that due to smoothing we observe period-t returns which are weighted sums of the true period-t return and k lagged past true returns. This means that, for $i = 0, 1, ..., k$, we have a process of the form

$$Q_t = \sum_{i=0}^{k} w_i R_{t-i},$$

where the weights $0 \le w_i \le 1$ satisfy the restriction $w_0 + w_1 + ... + w_k = 1$. Together with the condition that true returns are iid this makes sure that expected returns calculated on the basis of observed returns are an unbiased estimation for expected true returns. However, as a result of the smoothing process, period-t variance will be underestimated. In fact, according to our assumptions the following relationship will hold:

$$Var(Q_t) = Var(R_t) \sum_{i=0}^{k} w_i^2 \le Var(R_t).$$

Moreover, assume that we have two investments in two different private equity funds, n and l. Let $Corr(R_t^n, R_t^l)$ be the correlation coefficient between the unobserved true returns R_t^n, R_t^l, i.e. $Corr(R_t^n, R_t^l) = E[(R_t^n - E(R_t^n))(R_t^l - E(R_t^l))](Var(R_t^n)Var(R_t^l))^{1/2}$, and $Corr(Q_t^n, Q_t^l)$ be the correlation coefficient between the observed returns Q_t^n, Q_t^l, i.e. $Corr(Q_t^n, Q_t^l) = E[(Q_t^n - E(Q_t^n))\ [(Q_t^l - E(Q_t^l))](Var(Q_t^n)Var(Q_t^l))^{1/2}$, of the two funds under observation. Note that it would make sense in this model to assume that $Corr(R_t^n, R_s^l) = 0 \ \forall \ t \neq s$. In this case the following relationship must hold under our assumptions:

$$Corr\left(Q_t^n, Q_t^l\right) = \frac{\sum_{i=0}^{k} w_i^n w_i^l}{\sqrt{\sum_{i=0}^{k} \left(w_i^n\right)^2 \sum_{i=0}^{k} \left(w_i^l\right)^2}} Corr\left(R_t^n, R_t^l\right) \le Corr\left(R_t^n, R_t^l\right).$$

Unless the condition $w_i^n = w_i^l \ \forall i$ applies, contemporaneous correlation between observed returns of two different private equity funds is smaller than the correlation of true returns of the same funds. Hence, estimating multivariate distributions on the basis of observed returns subject to smoothing leads to underestimation of true return variance as well as true return correlation. Finally, as far as the correlation between observed returns and the return on the market portfolio is concerned, a bias could arise as well. Assume that R_t^M is the true, observable market portfolio return, for instance measured by an unbiased proxy like a market index. In this case we are interested in estimating the correlation coefficient between the true private equity return and the market index return, i.e. $Corr(R_t^M, R_t^l) = E[(R_t^M - E(R_t^M))(R_t^l - E(R_t^l))] (Var(R_t^M)Var(R_t^l))^{1/2}$. Again, given that we can only observe the smoothed proxy Q_t, only the following correlation can be measured: $Corr(R_t^M, Q_t^l) = E[(R_t^M - E(R_t^M))(Q_t^l - E(Q_t^l))](Var(R_t^M)Var(Q_t^l))^{1/2}$. Note that it would make sense in this model to assume that $Corr(R_t^M, R_s^l) = 0 \ \forall \ t \neq s$. Then, on the basis of our assumptions, we can show that due to smoothing correlation between returns on private equity investments and returns on public equity investments will be underestimated. In fact, the following relation will hold:

$$Corr\left(R_t^M, Q_t^l\right) = \frac{w_0^l}{\sqrt{\sum_{i=0}^{k}\left(w_i^l\right)^2}} Corr\left(R_t^M, R_t^l\right) \leq Corr\left(R_t^M, R_t^l\right).$$

To sum up, estimating private equity investment's return distributions on the basis of accounting-based appraisal values, like net asset values disclosed by general partners, is subject to serious estimation biases. In fact, Emery (2003) presented evidence in favour of so called stale pricing. In the context of a simple regression analysis he showed that NAV based private equity returns adjust with a lag movement to public returns. Of course, on the basis of the analysis presented here it will be possible to correct for this estimation bias, at least theoretically. Getmansky et al. (2003) show how such a corrected estimation can be achieved by using the returns on a large sample of hedge funds. However, it should be noted that this correction method rests on the assumption that the smoothing process does not affect the expected sample return and that the process, as described by the parameters $0 \leq w_i \leq 1$, is stable over time. As smoothing may not only be due to informational problems associated with illiquidity but also to deliberate actions set by general partners, it is still an open question whether the methods discussed here will really be sufficient for avoiding perceivable estimation biases. This is much more than a statistical issue, because even small estimation errors can have a large impact on asset allocation decisions. This issue will be treated in Section 3.

2.1.3 Noisy Smoothed Proxy Observations

In addition to the problems already mentioned in the preceding section, we would like to emphasize that the approach presented by Getmansky et al. (2003) also rules out observational noise. In other words, it may well be that even if the general manager of a private equity fund would really like to disclose the true *NAV* he is unable to do so because of the inability to infer the market value from available information. More formally spoken, the above model assumes that all observations Q_t are purely based on true returns and, hence, that accounting-based valuation exactly matches market valuation apart from its slower reaction to news.

In evaluating return distributions of private equity investments, alternative model specifications could be useful. Such models can allow for transitory deviations between the unobserved market value-based and the accounting-based returns. One possible specification would be to add observational noise to the above introduced equation for Q_t. In order to make our point in the simplest possible way, we first rule out the existence of any smoothing at all. In that case Q_t can be modelled as a noisy observation of the true return

$$Q_t = R_t + X_t,$$

where both returns are assumed to have identical unconditional expectations. The noise terms X_t have zero unconditional expectation, $E(X_t) = 0$, are uncorrelated with the true returns, $Corr(R_t; X_t) = 0$, and may exhibit linear dependence of the first-order autoregressive type

$$X_t = \rho X_{t-1} + \eta_t,$$

with $|\rho| < 1$ and iid innovations $\eta_t \sim N(0, \sigma^2)$. It then obviously follows

$$Var(Q_t) = Var(R_t) + Var(X_t) > Var(R_t).$$

Hence, although there is a possible effect of persistency in the observations Q_t, which is modelled through the dependent noise X_t, true return variance in this case is over-, not underestimated. It can easily be seen that one cannot rule out this result even under the assumption of smoothed returns. Even though an estimation of this model with observational noise may be performed within a state-space setting, this is potential bad news as to what our ability of identifying the true return distribution of a private equity investment is concerned.

To sum up, specifying a reliable model for Q_t might be a rather tricky task. And for the time being we cannot even be sure whether observed *NAV* based returns over- or underestimate true return variance.[6]

2.2 Cash Flow based Returns

Instead of relying on smoothed and/or noisy asset value based returns one can try to infer true investment returns from observable cash flow transactions between private equity funds and their limited partners. Under this perspective the return on a private equity investment could be measured by its internal rate of return (*IRR*). However, one should be careful in putting the *IRR* simply in relation to asset value based returns observed on the public equity market. This is because the *IRR* is a dollar-weighted return, while asset value based returns are time-weighted returns. In other words, while an asset value based return over a period of length *T* is simply the geometric mean of the single period realizations $1+R_t$, the *IRR* is a value-weighted average of these returns. Unless an investment consists of two cash flows only, a single initial investment and a single final repayment, the *IRR* would be different from the geometric mean of single period realizations $1+R_t$. The following simple example gives a flavour of the measurement relevance of this difference. Assume a private equity investment where – for whatever reasons– true market values are known and disclosed as *NAV*. The lifetime of the fund is assumed to be three years. Assume moreover that general partners define payouts to limited partners in a way that the cash flows are generated according to Table 2.2.

T	0	1	2	3
R_t		10%	20%	5%
NAV_t		110	32	33.6
CF_t	–100	0	100	33.6

Tab. 2.2. Unobservable true returns, true *NAV* as well as cash flows of a private equity fund investment.

6 As a corollary it should be noted that the model with observational noise generates an unbiased estimation for the true return correlation of two different private equity funds as well as for the return correlation of a private and public equity investment as long as the additional assumption $Cov(X_t^n; X_t^l) = 0$ holds.

Based on *NAV*, the average true return over the fund's lifetime is 11.5%,[7] while the *IRR* is 13.8%. Hence, observed *IRR* cannot be taken as an unbiased measure for the true expected return R_t .[8]

According to this simple insight an inference from observed *IRRs* on unobserved R_t is not obtained straightforwardly. However, an inference may be possible on the basis of additional assumptions. For instance, the following solution was proposed by Rouvinez (2003). In order to transform a multi-period cash flow stream into a two period wealth comparison he defines a start date T_0 and a terminal date T'. Any single private equity investment fund is raised a some point $t \geq T_0$ and liquidated at some point $T \leq T'$, with $t < T$. Now, all the cash paid into the fund is discounted back to period T_0 , while all the cash paid out to investors is compounded up to period T'. For this intertemporal transformation the risk free interest rate r is used; it is assumed to be constant. Hence, initial and terminal wealth corresponding to cash flows generated by a single private equity fund can be calculated as follows:

$$W_{T_0} = \sum\nolimits_{i=t}^{T} CF_i^{in} (1+r)^{-(i-T_0)},$$
$$W_{T'} = \sum\nolimits_{i=t}^{T} CF_i^{out} (1+r)^{T'-i}.$$

Here, CF_i^{in} resp. , CF_i^{out} is the cash paid into or paid out of the fund in period i. The ratio of terminal wealth to initial wealth, basically, gives an information into how many Euros a one Euro investment at time T_0 will be transformed up to time T', given that the investment is exposed to private equity risk for a total period of $T–t$. Now, the expected true return during the exposure to private equity risk, i.e. $E[W_T/W_t]$, can be expressed as a function of time length of this exposure, i.e. $T–t$, and the expected overall rate of return, i.e. $E[W_{T'}/W_{T_0}]$. In fact, it can be shown that the following relationship must hold:

[7] It should be noted that the drop in the *NAV* from period 1 to period 2 is due to the payout of 100. Hence, the return has to be measured on the basis of a payout corrected *NAV*.

[8] For the *IRR* to be an unbiased estimator, R_t must be iid and only two cash flows are allowed to occur over the lifetime of the fund.

$$\ln\left(E\left[\frac{W_T}{W_t}\right]\right)=\ln\left(E\left[\frac{W_{T'}}{W_{T_0}}\right]\right)+\left(T-t-T'+T_0\right)\ln(1+r),$$

$$Var\left(\frac{W_T}{W_t}\right)=\ln\left(1+\frac{Var\left(\frac{W_{T'}}{W_{T_0}}\right)}{E\left[\frac{W_{T'}}{W_{T_0}}\right]^2}\right).$$

Using real life cash flow figures this expectation and variance can easily be calculated. In fact, $E[W_T/W_t]$ can be regarded as the expectation of a pooled rate of return of the whole asset class. According to the approach used by Rouvinez (2003) it is supposed to be an estimation for the true return realised in the private equity industry, as he assumes the relation $R_{T-t} = (W_T/W_t)^{1/(T-t)}-1$ to hold. For the same reason, therefore, the second equation yields the true return variance.

This approach has several drawbacks. First, we will get only an estimation for an average rate of return over a longer period of time. Hence, no dependency with market movements can be detected here. Second, this approach is not unbiased as the return measured over the private equity exposure time depends on how the true return generating process relates to risk free interest rates. One can see this problem very quickly by looking at the example presented above. Assume, for instance, that the risk free rate is 10%. In this case, applying the calculation proposed by Rouvinez would generate an average rate of return of 12.8%, which is higher than the true average rate of return. On the other side, using a risk free rate of 0% would lead to an average rate of return of 10.1%, which is again a biased result. Third, this approach does not allow for estimating correlations within the asset class.

Some of this criticism can be circumvented by using an approach introduced by Chen et al. (2002), although this approach has some caveats as well. They start from the presumption that for every single fund the relation $W_T/W_0 = (1+IRR)^{1/T}$ holds, where for simplicity we assume $t = 0$. Now, this return is put into relation with the returns on the public equity market over the same period. Therefore, the following modified market model holding for every single fund i is specified:

$$T_i \ln(1+IRR_i)=\beta\ln\left(1+R^M_{T_i-0}\right)+\varepsilon_i.$$

Here, $R_{Ti-0}{}^{M}$ denotes the public market return, as measured by a representative index over fund's i lifetime, and β is the elasticity of fund returns to the market return common to all private equity funds. Moreover, it is assumed that $\varepsilon \sim N(\alpha T, \sigma^2 T)$ and $Cov[\varepsilon_i, \varepsilon_j] = \rho\tau_{ij}\ \sigma^2$ holds; here α is a fund specific return component per unit of time common to all private equity funds, ρ is the per unit of time correlation of the non market driven part of all distinct pairs of funds' returns, and τ_{ij} is the coexistence time of two funds i and j. Now, Chen et al. (2002) show that under these assumptions a maximum-likelihood estimation for the asset class specific parameters α, β, ρ and σ can be derived.

As mentioned, this approach has some drawbacks as well. The major one is the fact that assuming all cash flows paid out by a private equity fund to be reinvested at the *IRR* up to terminal date T is not correct. From the numerical example above one can easily see that this would overestimate the true return of the fund. Of course, in general it is not clear whether this approach yields an over- or an underestimation of true funds' returns. In any case, however, the estimation would be biased.

In order to circumvent these problems, we outline an idea for a third approach, not yet discussed in the literature. Assume that from an ex-ante perspective the true return process R_t can be explained by the CAPM. In that case the conditional expectation of R_t can be written as

$$E\left[R_t \left| R_t^M \right.\right] = r_t(1-\beta) + \beta R_t^M,$$

where r_t is the risk free interest rate governing in period t. Now, for every liquidated fund the following equation –using the conditional expectation of R_t– must hold:

$$-CF_0 = E\left\{ \sum_{t=1}^{T} CF_t \ (1+IRR)^{-t} \right\} = E\left\{ \sum_{t=1}^{T} CF_t \left[\prod_{i=1}^{t} \left(1 + r_i(1-\beta) + \beta R_i^M \right) \right]^{-t} \right\}.$$

It is a question to be left open here, whether this ex-ante equation can be transformed in an ex-post equation in a way that it will be possible to make an efficient and unbiased estimation for the parameter β governing the return process of the whole asset class. From this we can finally assess the distribution of the return process, as the following must hold:

$$Var(R_t) = \beta^2 Var(R_t^M) + Var(\varepsilon_t),$$
$$Corr(R_t^M, R_t) = \beta \sqrt{\frac{Var(R_t^M)}{Var(R_t)}}.$$

Obviously, an estimation of cross correlation of private equity investment returns cannot be derived in the context of this approach, as we assume them to be from one single distribution. The most important advantage of this approach is the fact that we do not need any kind of reinvestment hypothesis with respect to a fund's cash flows. Therefore, no potential bias is induced due to the lack of any kind of reinvestment assumption.

3. Risk Management and Asset Allocation

In the preceding section we showed how an assessment of distributional characteristics of true returns R_t could be derived from observed proxy returns Q_t, or observed *IRR*s. Such inference included the first and second moment of the return distribution F, i.e. the expected return on a private equity investment $E(R_t)$ as well as the return variance $Var(R_t)$. Moreover, under certain circumstances we were able to make an inference with respect to the correlation of private equity returns with other asset classes $Cov(R_t^M; R_t^{'})$ as well as with other investments in the same asset class $Corr(R_t^n, R_t^l)$.

Once distributional information is gathered from observed returns, most of the commonly used risk management as well as asset allocation techniques can be applied to portfolios containing private equity investments. This, for instance, is especially true with respect to the Value-at-Risk (VaR) approach, where the task is to infer some p-quantile, $q_p = F^{-1}(p)$, of the distribution F of the returns. We point out, however, that the particular features of private equity cast some doubt on solely applying traditional risk management as well as asset allocation techniques. Empirical results on private equity and alternative investment such as venture capital indicate large standard deviations of period returns as well as significant skewness and excess kurtosis in the return distribution. This would have a particular impact on risk management, which we will discuss in the following section.

3.1 Specific Issues in Risk Management

First, it should be noted that the asset value based estimation approaches presented in Section 2.1 rely on the assumption that smoothing is kind of a stationary process. Especially, we ruled out that this process is driven by changing strategic goals of reporting by general partners. In practice, however, one cannot rule out that the degree of smoothing is related to incentives governing the behaviour of general partners and, hence, will change over time depending on conditions not reflected in the model. For instance, one might presume that adjustment to market prices, given that they are privately known by general partners, is faster when they are increasing, especially when the whole market is in a positive mood, and slower when they are decreasing. Therefore, the presumption that the estimated distribution F already integrates illiquidity effects in a sufficient manner should not be taken as granted.[9] The important consequence for risk management is that F is not a stationary distribution. Unfortunately, no reliable empirical information is available in this regard as the techniques for integrating this kind of problem in the estimation of F have still to be developed.

Second, as long as we try to infer the true return distribution by looking at a fund's cash flow it is rather unclear as to what extent a liquidity discount is then taken into account. One may presume that an investor forced to sell a stake in a private equity fund prematurely faces an *IRR* which is considerably lower than the *IRR* generated without premature liquidation. This is an important point, because the expected returns derived under this methodology integrate an illiquidity driven risk premium. This is an important aspect that has to be taken into account, especially for such groups of institutional investors that may face severe liquidity shocks.

Third, it is well known from the risk management literature that asset return's distributions are not fully captured by the assumption of normality and independence. Special emphasis has been put in this context on the empirical regularity of fat tails, i.e. the phenomenon that extreme realisations happen more frequently than predicted under a normal distribution. As risk management is focused on extreme realisations, this is one of the most important theoretical and practical challenges. In fact, Ljungqvist and

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[9] Of course, also the traditional risk management literature dedicated some attention to illiquidity issues; cf. for example Jorion (2001, Chapter 14). However, as these authors looked at traded assets, liquidity costs could be measured by bid-/ask-spreads, for instance. This would not be possible in the context of private equity.

Richardson (2003) and Kaplan and Schoar (2003) report that cash flow based private equity fund's returns are heavily skewed in the sense that there is a significant downside in the form of funds performing poorly on a relative basis. However, Cochrane (2001) found a much less pronounced skewness, if one switches from arithmetic to geometric returns of single venture capital transactions. From a theoretical perspective one might expect returns to be skewed because of the option-like payoff structure of risky claims.[10] This skewness should be more pronounced the higher the debt is relative to a firm's market value and the more the total firm value can be modelled as an option-like payoff itself. The last point, at least, gives a strong indication that return skewness of young and innovative business ventures should be more pronounced, as their investment projects often can be characterized as a real option. Therefore, the implementation of extreme value theory in risk management tools may be especially important for investors exposed to private equity risks. The bad news are, however, that for the time being we do not have an empirically well founded understanding of extreme value behaviour in private equity investments.

Fourth, non-normality features of private equity portfolio returns may also relate to what a particular institutional investor defines as a proper investing strategy. The choice of such a strategy will depend on the institutional investor's financial goals as well as on his particular knowledge advantages. Two common generic strategies are diversification and specialisation. While diversification lowers risk as long as asset returns are not perfectly correlated and increases the degree of normality from a financial risk management point of view, specialisation does not. When following a diversification strategy investors seek constrained risk reduction for their overall asset portfolio. This strategy includes diversification not only between companies and industries but also between financing stages especially when venture capital investments are considered. When following a specialisation strategy with private equity investments, investors increase overall risk relative to diversification. The payoff from exposing a portfolio to diversifiable risk is that it may offer rents from controlling activities. This is especially important for institutional investors acting as general partners, as there is an obvious principal-agent relationship between the venture capital investor and its investee.[11]

[10] It should be noted that financial theory models equity as a contingent claim –a call option– on firm assets. It predicts stronger nonlinearity of payoffs for out of the money call options. For related empirical findings based on traded IPO aftermarket equity issues in the German Neuer Markt see for example Wagner (2001).

[11] Cf. in this regard Reid et al. (1997).

Moreover, high degrees of specialisation can be helpful in building up reputation and further in gaining access to networks, to information flows as well as to deal flow from other private equity investors. In a survey study of venture capital investors, Norton and Tenenbaum (1993) give evidence that some considerable number of investors in their sample follow such specialisation strategies. Of course, an empirical well-founded understanding of the specific characteristics of private equity returns, especially as far as extreme value realisations are concerned, is even more important for institutional investors following such a specialisation strategy.

3.2 Specific Issues in Asset Allocation

Taking the aforementioned methodological problems in estimating the parameters of the distribution F into account, one may not be surprised by the quite different results obtained in the literature. As far as asset value based approaches are concerned, for the time being, there are two studies to be mentioned here. Emery (2003), whose results should be interpreted cautiously as he did not explicitly correct for the smoothing problem, reports that returns calculated on the basis of biannual *NAV* US-funds data average to 15% for LBO-funds and 25% for venture capital funds per year.[12] The correlation with S&P500 returns is 56% resp. 64%. The correlation between LBO- and venture capital funds' returns is almost zero. Kaplan and Schoar (2003) use a large data set provided by Venture Economics. They try to overcome the smoothing problem by looking only at funds which have already been closed or have been alive for at least five years. In fact, they can show that the correlation between the rates of return calculated for this subsample of funds is highly correlated to the *IRR* of the same subsample. Due to this restriction, however, the approach of Kaplan and Schoar (2003) is, in fact, not that different from a cash flow based approach. For a sample of more than 1'000 funds they find an average *IRR* of 17%, while the median is 12%. The standard deviation of the *IRR* is 32%. Moreover, they find evidence in favour of performance persistence.

As far as other cash flow based approaches are concerned, Ljungqvist and Richardson (2003), for instance, find a median *IRR* of almost 20% for a US dominated sample of 73 funds with a cash flow history of at least nine years; the standard deviation of the *IRR* is

[12] The reason why Emery (2003) calculates the returns on a biannual basis relates to the smoothing problem. In fact, he starts from the plausible presumption that smoothing effects vanish in the long run. However, whether a two year return period is already sufficient in order to overcome the smoothing bias is a question with not clear cut answer.

22%. Cochrane (2001) uses data from more than 16'000 single venture capital transactions and calculates an arithmetic return of 59% with a standard deviation of 100%. More interesting data, at least from an asset allocation perspective, has been reported by Chen et al. (2002). By using *IRR* data of about 150 liquidated venture capital funds and by applying the maximum likelihood estimation technique explained in Section 2.2, they calculate an average venture capital fund return of 45%, with a standard deviation of 116% and a correlation with large capitalization stock's returns of 4%. Finally, Rouvinez (2003) by applying a cash flow based method already explained in Section 2.2 finds an expected return on a private equity investment of 14% and a standard deviation of 34%. His methodology does not allow for calculating correlation coefficients with public market returns. Of course, these results may still be interpreted cautiously as they have been derived despite of severe data restriction problems. It will therefore still take a couple of years until reliable empirical results will be available. Nevertheless, it may become something like a stylized fact that venture capital returns are perceivably higher than returns on non-venture capital private equity investments. This seems to come along with a very much higher volatility as well as with a lower correlation to public equity returns. Given that future empirical research will corroborate this result, this will become an important issue in asset allocation decisions.

$\sqrt{Var(R_t)}$	$Corr(R_t^M, R_t^I)$	α	portfolio return	portfolio standard deviation
		0,0%	8,0%	15%
100%	5%	3,0%	8,7%	15%
100%	10%	1,5%	8,3%	15%
50%	5%	14,2%	11,1%	15%
50%	10%	11,7%	10,6%	15%
50%	20%	6,2%	9,4%	15%
30%	30%	21,0%	12,6%	15%

Tab. 3.2. Optimal portfolio weights of a private equity asset in a private and public equity portfolio for different distributional parameters.

Finally, we would like to emphasize that the lack of clear cut empirical results with respect to the conditional and unconditional distribution of private equity investment returns is a serious problem making any asset allocation decision a rather tricky task. In fact, even a slight shift in the distributional parameters may have a very large impact on portfolio allocation. Hence, even a slightly biased assessment of these parameters could lead to dramatic errors in asset allocation. This becomes clear from the next table, where we show how the optimal weight of a private equity investment α in an equity portfolio changes according to a change in the return variance as well as in the return correlation with public equity. In order to calculate these weights we assumed that the expected return on a public investment is 8% with a standard deviation of 15%. Moreover, we assumed the expected return on a private equity investment to be 30%. The weights for the private equity asset were derived by maximizing the expected portfolio return under the constraint that portfolio variance is equal to the variance of a 100% public equity portfolio. Other asset classes, like bonds or real estate, are not taken into account here.

4. Conclusion

Private equity has become an increasingly important alternative asset class for institutional investors as it may offer return as well as diversification benefits relative to traditional stock and bond market investments. Despite the downturn of the industry over the years 2001 to 2003, it is commonly believed that private equity will become an even more important source of corporate financing over the years to come. For this reason, understanding and managing risks associated with this asset class is of crucial importance for institutional investors.

This article aims at improving this understanding and, hence, to give a foundation for solving specific problems arising in the context of private equity risk management and asset allocation decisions. As a starting point, we emphasized that –in our view– the most specific characteristic of private equity is the lack of an organised secondary market. Hence, investing comes along with low liquidity or even illiquidity, i.e. with the impossibility to transact at a targeted point in time and/or with the occurrence of substantial transaction costs.

There are two major consequences of the lack of an organised secondary market that have been treated extensively in this paper. Firstly, illiquidity implies that it is not possible to observe a continuous series of true investment returns over time. In other words, illiquidity goes along with serious performance measurement problems. Section 2 was entirely devoted to this problem in that we tried to show to what extent this sort of measurement problem can be resolved. Basically, there are two ways for doing this. Either one tries to infer market values from reported asset values, or one tries to infer true investment returns from realized cash flow based investment returns. As we showed, both approaches are not free of serious restrictions. As a consequence, our empirical understanding of the risk-/return characteristics of this asset class is still incomplete and should be subjected to further research. Secondly, given that we would be able to overcome these measurement problems we could apply several well-known risk management as well as asset allocation methods to the private equity asset class as well. However, in our view there are some specific issues in this context that apply solely to portfolios exposed to private equity risk. We discussed these issues in Section 3.

As far as risk management is concerned, specific issues relate to the following problems. Reliability of return distribution measurement is a rather serious problem, as one cannot rule out general partners to follow a strategic disclosure policy, which would be very difficult to integrate in a statistical model of a return generating process. It is also unclear whether a cash flow based return inference model would really capture the whole return impact of illiquidity. Moreover, there are good theoretical reasons suggesting that private equity returns, especially when they are related to venture capital investments, will be governed by a distribution with much more pronounced fat tails than public equity returns. Finally, these issues may be faced in a different way depending on whether the investor follows a diversification or specialization strategy. There are some preliminary empirical results indicating that specialization plays a much more important role in the private equity industry than in the public equity industry. All these issues enrich risk management of portfolios exposed to private equity risk with rather specific problems.

Beyond these risk management issues, we also discussed issues related to asset allocation decisions. Our major point here was to show that the empirical understanding of the risk-/return characteristics of the private equity class is, in fact, incomplete and, to a certain extent, contradictory. This is important as purposeful asset allocation decisions can only be based on a well-founded empirical understanding of risk-/return characteristics. Moreover, we showed that even slight biases in the estimated distributional parameters can have a large impact on asset allocation decisions. This is one of the major reasons why we strongly emphasize the need for much more additional empirical and theoretical work on this asset class.

References

Campbell, J.Y. / Lo, A.C., MacKinlay, A.C. (1997): The Econometrics of Financial Markets, Princeton University Press, Princeton.

Chen, P. / Baierl, G.T. / Kaplan, P.D. (2002): Venture Capital and its Role in Strategic Asset Allocation, in: Journal of Portfolio Management, Winter 2002, p. 83-89.

Cochrane, J.H. (2001): The Risk and Return of Venture Capital, NBER Working Paper No. w8066.

Cohen, K.J. / Hawawini, G.A. / Maier, S.F. / Schwartz, R.A. / Whitcomb DK (1983): Friction in the Trading Process and the Estimation of Systematic Risk, in: Journal of Financial Economics, 12, p. 263-278.

Emery K (2003): Private Equity Risk and Reward: Assessing the Stale Pricing Problem, in: Journal of Private Equity, Spring 2003, p. 43-50.

Getmansky, M. / Lo, A.W. / Makarov, I. (2003): An Econometric Model of Serial Correlation and Illiquidity in Hedge Fund Returns, NBER Working Paper.

Gompers, P. / Lerner, J. (1999): The Venture Capital Cycle, MIT Press, Cambridge.

Jorion, P. (2001): Value at Risk, McGraw-Hill, New York et al.

Kaplan, S. / Schoar, A. (2003): Private Equity Performance: Returns, Persistence and Capital Flows, NBER Working Paper No. w9807.

Ljungqvist, A. / Richardson, M. (2003): The Cash Flow, Return, and Risk Characteristics of Private Equity, NBER Working Paper No. w9454.

Lo, A. / MacKinlay, A.C. (1990): An Econometric Analysis of Nonsynchronous-Trading, in: Journal of Econometrics, 45, p. 181-212.

Longstaff, F. (1995): How much can Marketability affect Security Values?, in: Journal of Finance, 50, p. 1767-1774.

Norton, E. / Tenenbaum, B.H. (1993): Specialization versus Diversification as a Venture Capital Investment Strategy, in: Journal of Business Venturing, 8, p. 431-442.

Peng, L. (2001): A New Approach to Valuing Illiquid Asset Portfolios, Yale ICF Working Paper.

Reid, G.C. / Terry, N.G. / Smith, J.A. (1997): Risk Management in Venture Capital Investor-Investee Relations, in: European Journal of Finance, 3, p. 27-47.

Roll, R. (1981): A Possible Explanation of the Small Firm Effect, in: Journal of Finance, 36, p. 879-888.

Rouvinez, C. (2003): How Volatile is Private Equity?, in: Private Equity International, June 2003, p. 22-24.

Schilit, W.K. (1993): A Comparative Analysis of the Performance of Venture Capital Funds, Stocks and Bonds, and other Investment Opportunities, in: International Review of Strategic Management, 4, p. 301-320.

Scholes, M. / Williams, J. (1977): Estimating Betas from Nonsynchronous Data, in: Journal of Financial Economics, 5, p. 309-328.

Wagner, N. (2001): A Market Model with Time-Varying Moments and Results on Neuer Markt Stock Returns, SSRN Working Paper.