Equilibrium Security Prices with Capital Income Taxes and an Exogenous Interest Rate*

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We are interested in the effect of capital income taxes upon security prices when investors face locally segmented stock markets and a global bond market. Therefore, we analyze an equilibrium model of an economy with binomial uncertainty, an exogenous risk-free interest rate and a representative stand-in household. In this setting, the pricing effect for domestic securities is shown to be a function in three determinants: the covariance between pre-tax payoffs of securities and the aggregated market portfolio, the exogenous pre-tax interest rate and the effect of taxation (and redistribution) on the aggregate welfare of the stand-in household. We find that taxation of capital income is non-distorting if tax proceeds are immediately redistributed within the cohort of capital market participants. If, however, taxation represents a policy tool to transfer wealth from capital market participants to non-market participants, the level of the statutory tax rate is reflected in equilibrium security prices and taxation affects households portfolio decisions, which in turn may affect investment decision of firms.

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1. Introduction

In most economies of the world capital income is taxed at the personal level.[1] This causes two effects. First, it drives a wedge between two dominant building blocks of any economy: investors and firms. Second, it allocates funds to the public sector. Accordingly, it seems natural to ask about the effect of a capital income tax and the corresponding tax proceeds upon security prices, which in turn determine the cost of capital for firms as well as (future) consumption possibilities of investors. Surprisingly, however, there are relatively few theoretical works discussing the effect of taxes upon the level of security prices, which we will call the *pricing effect* of taxation.

The current paper aims to narrow this gap by studying the pricing effect of a linear tax on investors' interest income, dividend income and capital gains.[2]Thereby, three things are worthwhile to note. First, since capital income taxes produce uncertain tax revenues for the public authority, the analysis must consider taxation and expenditure of the authority simultaneously. Subsequently, the aggregate of the tax code and authority's expenditure program is called *policy design*. Second, the pricing effect may only be expected in an economy where investors exposed to taxation of capital income substantially affect security prices. Put another way, the pricing effect will not be observed in a small economy with perfectly integrated capital markets. However, empirical evidence indicates that capital markets, and in particular stock markets, in general are not perfectly integrated.[3] Thus, our model assumes that domestic stocks are traded in a locally segmented stock market while investors face a global bond market. In this setting, which we will call *semi-closed*, the global bond market offers an exogenous risk-free interest rate to domestic investors. The market price of risk, in contrast, is determined endogenously in the domestic stock market. EMU-countries with an independent European Central Bank controlling the interest rate for (risk-free) Euro-investments may serve as a straightforward rationale for our semi-closed model set-up. Third, recall the well

[1]See OECD (1994) for an introduction to capital income tax regimes of many developed countries. Joumard (2001) and Schratzenstaller (2003) discuss taxation of capital within the European Union.

[2]Essentially, such a tax equals a flat withholding tax on capital income including dividends, interest, and capital gains as will be introduced in Germany from 2009 onwards (see Bundesfinanzministerium (2006) and for more details Bundesrat (2007)).

[3]For instance, econometric analyses of investment decisions find that investor behavior in stock markets is characterized by a *home bias* (e.g. Lewis, 1999) and even a *local-bias* (e.g. Coval and Moskowitz, 1999; Hong, Kubik and Stein, 2007).

documented empirical fact of limited market participation pioneered by Mankiw and Zeldes (1991).[4] Thus, our analysis assumes two different groups of domestic individuals: capital market investors, subsequently called *insiders*, and individuals not investing in capital markets (*outsiders*). Obviously, in the absence of taxation only insiders determine the market price of risk on the stock market. Accordingly, our theoretical model considers only insiders and their behavior when determining the pricing effect. However, arguing that the large cohort of outsiders may vote for capital income taxes as a tool to re-allocate wealth within the society, we are in particular interested in the effect of capital income taxes in the case that the authority distributes tax proceeds to outsiders.[5]

The analysis is restricted to a single-period binomial model, which allows us to derive closed form results for a broad variety of von Neumann/Morgenstern preferences. We start by characterizing the price of any security as the state-price weighted sum of its state-dependent post-tax payoffs. This enables us to disentangle the pricing effect of any policy design into two sub-effects. First, a policy design may alter equilibrium state prices. This effect, which we call the *equilibrium effect*, essentially mirrors the impact of the policy design upon the well-being of the stand-in household representing domestic insiders. Second, we call the effect upon post-tax payoffs promised by a particular security the *payoff effect.* Throughout, pre-tax payoffs of securities are exogenous primitives to our analysis. While this assumption simplifies our analysis considerably, it does not alter our qualitative results (as long as domestic production functions are continuous). Putting the two effects together gives us the pricing effect. It is shown that the pricing effect for a domestic security is a function of three determinants: (i) the covariance between the pre-tax payoffs of the security and the pre-tax payoff of the market portfolio, (ii) the exogenous pre-tax risk-free rate, and (iii) the tax effect for risk-neutral probabilities of the domestic stock market. Note that only the third determinant may be sensitive to the authority's tax (and redistribution) regime.

The rest of the paper is organized as follows. Section 2 reviews related literature.

[4]The limited market participation effect describes the empirical observation that only a (small) fraction of households participates in capital markets. Market frictions like information and transaction costs are often used to rationalize limited market participation (e.g. Guiso, Haliassos and Jappelli, 2003; Alan, 2006). Polkovnichenko (2004, fn. 6) points out some sources of transaction costs: "*the direct cost of maintaining an equity account with a broker or mutual fund, additional time spent filing taxes, the cost of learning about equity investments or paying for professional portfolio advice*".

[5]This argument seems particularly interesting, if one keeps in mind that market participation is generally positively correlated to household wealth (e.g. Guiso, Haliassos and Jappelli, 2003).

Subsequently, section 3 presents the general framework in the absence of taxation. Section 4 extends the model for taxation and section 5 examines the pricing effect assuming that tax proceeds are completely transfered to outsiders. Finally, section 6 presents the conclusion.

2. Related Literature

Analyzing the effects of taxation has a long history in economic literature. However, as noted by Poterba (2002, p. 1161) the pricing effect of capital income taxation has not received much attention until now. In particular, the major strand of literature examining economic effects of capital income taxes, the *public economics literature*, does not examine this pricing effect. In contrast, it seems to be mainly concerned with the effect of capital income taxes upon the risk-taking behavior of individual agents facing exogenous security prices. The *asset pricing literature*, being the second most important line of literature analyzing the effect of taxation, is mainly interested in the effect of capital income taxes upon the prevailing risk-return structure and does not account for the fact that taxation allocates funds to the public sector.

Starting from the seminal work of Domar and Musgrave (1944) much of the public economics literature concentrates on the effects of taxation upon the risk allocation process (e.g. Mossin, 1968, Stiglitz, 1969, Sandmo, 1989, Hilgers and Schindler, 2004 among others). The authors examine saving decisions and portfolio choice problems of individual investors facing exogenous pre-tax security returns. Sandmo (1989) for instance finds that the taxation of capital income does not induce any substitution effect from assets with low risk to assets with high risk in a small open economy. However, these models either represent partial equilibrium models or models implicitly relying on the small open economy set-up. Our analysis reveals that in a semi-closed economy the taxation of capital income may induce effects not observed in a small open economy. For instance, we find that a policy regime which levies capital income taxes and distributes tax revenues to outsiders induces a substitution effect towards securities that are positively correlated with the market portfolio.

The asset pricing literature generally discusses the impact of capital income taxes in a closed economy framework. Concentrating on the prevailing risk-return structure, the analysis in general relies on mean-variance preferences or (multivariate) normal distributed security returns (e.g. Brennan, 1970; Litzenberger and Ramaswamy, 1979, 1980 and others). In these kind of models the market price of risk is a non-trivial function in agents' coefficient of global absolute risk aversion (e.g. Rubinstein, 1973), a fact that makes it virtually impossible to derive analytical results for the effect of taxation upon equilibrium security prices for reasonable preferences (e.g. CRRA preferences).[6] Restricting the analysis to a binomial model and applying the state-price pricing approach allows us to analyze the effect of capital income taxes on the *level* of equilibrium security prices.

Finally, three papers examining the effects of taxation in general equilibrium models of closed economies are worth discussion in some detail here.[7] These papers are examples of a strand of literature that is generally interested in *neutrality results*, i.e. conditions ensuring that taxation of capital income does not distort economic decisions within an economy. Mintz (1982), for instance, analyzes a corporate tax code that is basically equivalent to a personal tax code on excess returns. The author shows that neglecting general equilibrium effects, i.e. effects upon agents' marginal rate of intertemporal substitution and therefore upon the risk-free rate and the market price of risk, the tax code is neutral.[8] Gordon (1985) analyzes a tax code comprising property, corporate, and personal taxes. The author shows that investment decisions are unaffected by taxation, if (i) there is no tax revenue from risk-free investments and (ii) transfer payments leave any agents' wealth position unaffected. Given that tax revenues for risk-free investments must be equal to zero, the neutral tax system of Gordon (1985) is essentially equivalent to an excess return tax.[9] Directly analyzing a personal tax on excess returns Konrad (1991) shows that such a tax rate is neutral even in a heterogeneous investor economy allowing for endogenous production and arbitrary, budget-balancing transfer payments. Our analysis shows that in a semi-closed economy even a flat capital income tax on interest income, dividend income and capital gains may be neutral

[6] Similar problems appear in the analysis of Auerbach and King (1983).

[7] The analysis of McGrattan and Prescot (2005) is not discussed here, since it relies on a deterministic growth model and thus does not allow to study the effect of capital income taxes on the market price of risk.

[8] Allowing for non-state contingent transfer payments and shared public goods, Mintz (1982, Lemma 1) provides – rather strict – conditions ensuring that there are no general equilibrium effects for a particular firm.

[9] In a framework similar to the one examined by Gordon (1985), the analysis of Bulow and Summers (1984) points out that the equilibrium effect of taxation significantly depends upon the fact whether or not taxation cuts in gains and risk symmetrically.

with respect to equilibrium outcomes. This neutrality result however only holds if in our semi-closed model set-up tax proceeds are immediately and fully rebated to insiders and these insiders treat redistribution as a perfect substitute for post-tax capital income.

3. The Model Without Taxes

Consider the following single-period model of an economy inhabited by m consumers. At the outset of the period ($t = 0$), two frictionless capital markets open: a domestic stock market and a global market for risk-free investments, subsequently labeled *bond market*. Only domestic investors are allowed to hold securities traded in the domestic stock market. In contrast, risk-free securities traded in the global bond market may be held by investors worldwide.[10] Essentially, in the single-period set-up analyzed here there is only one bond traded in the global bond market and the impact of domestic investors on the price of the single-period bond is supposed to be zero. In both markets investors may trade securities free of transaction costs. At the end of the period ($t = 1$), all securities yield payoffs in monetary units of account.

Further, domestic consumers group into two categories: n insiders, which participate in the domestic stock market as well as in the global bond market and $m - n$ outsiders, which abstain from participating in these markets. This segmentation is considered exogenous to our model. Insiders own a portfolio of domestic securities and a position in the global bond prior to the beginning of the period. In time-0 they engage in both financial markets and trade securities in order to maximize their utility over monetary time-1 income. By assumption security payoffs are the only source of time-1 income for market participants.

Furthermore we assume that there exists a single *virtual household* such that if this household is endowed with aggregate resources of all market participants, then equilibrium security prices are characterized by the household's optimization problem (e.g. Duffie, 1996, chapter 1). In general, preferences of this *pricing household* are a function of the level and the structure of the initial resource distribution within the economy. We shall assume, however, that the pricing household is independent of the level and structure of initial resources within the cohort

[10] We do not consider the effect of exchange rates and the associated uncertainty in our analysis.

of market participants, i.e. we presume that there exists a representative *stand-in household* that throughout mirrors the economic behavior of insiders (not only in equilibrium).[11] The following assumptions 1 to 4 specify our model in more detail.

Assumption 1 (Timeline)**.** *Time-0 is certain, whereas in time-1 one of two states* $\{r, b\}$ *realizes. The (subjective) probability that state* $s(\in \{r, b\})$ *occurs is denoted as* $\phi_s \in (0, 1)$.

Assumption 2 (Frictionless global bond market)**.** *There is a frictionless global bond market, where investors may trade in a single-period risk-free bond with an exogenous time-0 price* $p_0 = 1$ *and a risk-free time-1 payoff* $(1 + r_0) \geq 1$.

The risk-free bond of the global bond market yields an exogenous non-negative risk-free interest rate r_0. Accordingly, the insiders' investment universe would be complete (i.e. span both states) with only a single risky security traded in the domestic stock market. However, since we are interested in the effect of taxation on different classes of domestic securities, we allow for a variety of K securities in the domestic stock market.

Assumption 3 (Frictionless domestic stock market)**.** *There is a frictionless domestic stock market, where only domestic investors are allowed to trade.* K *securities, which are all in a net supply of one, are traded in this market. In* $t = 1$ *these securities offer exogenously given state-dependent payoffs* $z_k = (z_{kr}, z_{kb})$ *(measured in units of monetary account). For the payoff of the market portfolio* $M_s = \sum_k z_{ks}$ *we assume* $0 < M_r < M_b$. *Accordingly, we call* r *and* b *'recession' and 'boom' state, respectively.*

Assumption 4 (Representative household)**.** *There is a stand-in household with preferences over (monetary) time-1 income* Z *that may be represented by* $U(Z) = \phi_r \times u(Z_r) + \phi_b \times u(Z_b)$, *where* u *is twice-differentiable with* $u' > 0$ *and* $u'' < 0$. *Aggregate resources of the stand-in household equal aggregate resources of insiders, i.e. its monetary time-1 income is given by the time-1 payoff of the market portfolio* $M_1 = (M_r, M_b)$ *plus the payoff of insiders' aggregate position in the global bond* $Y_1 = Y_0 \times (1 + r_0)$.

Equilibrium security prices are now characterized by the stand-in household's optimization problem subject to the following constraints: (i) $0 \leq Y_0 + \sum_{k=1}^{K} p_k$

[11] This assumption is satisfied if our model economy allows for the *aggregation of preferences*, i.e. if equilibrium security prices are independent of the distribution of initial wealth within the economy. For a dynamic economy Rubinstein (1974) reports sufficient conditions for aggregation of preferences. Brennan and Kraus (1978) prove them to be necessary.

and (ii) $Z_s = M_s + Y_1$ for $s \in \{r; b\}$.[12]

Subsequently, we will make use of the following definition: $X_s = M_s + Y_1$ for $s \in \{r; b\}$. Given the stand-in household's optimization problem standard arguments imply the following representation of the equilibrium price of security k

$$\frac{p_k}{\pi_0} = \sum_{s \in \{r;b\}} \phi_s \times \frac{u'(X_s)}{\mathbf{E}[u'(X_1)]} \times z_{ks}, \tag{1}$$

where π_0 is a *normalization parameter* and $\mathbf{E}$ denotes the expectation operator with respect to the probability measure ϕ (e.g. Duffie, 1996, chapter 1). Moreover, let π_s denote the *equilibrium state price* (ESP) for state s, i.e. the price of a primitive security offering one unit of monetary account in time-1 if (and only if) state s occurs and nothing otherwise. With equation (1) the sum of the two ESPs equals the normalization parameter π_0. Furthermore, applying equation (1) to the global bond gives $1/(1 + r_0) = \pi_r + \pi_b = \pi_0$. Hence, the ESP of state s is given by $\pi_s = (1 + r_0)^{-1} \times \{u'(M_s)/\mathbf{E}[u'(M)]\} \times \phi_s$ and the equilibrium price of security k is

$$p_k = \frac{1}{1 + r_0} \times \sum_{s \in \{r;b\}} \phi_s \times \frac{u'(X_s)}{\mathbf{E}[u'(X)]} \times z_{ks} \tag{2}$$

The ESPs are strictly positive and the set of *normalized ESPs* $q_s = (1 + r_0) \times \pi_s = \{u'(X_s)/\mathbf{E}[u'(X)]\} \times \phi_s$ defines a probability measure Q on the state space $\{r; b\}$. Thus, $p_k = (1 + r_0)^{-1} \times \mathbf{E}^Q[z_k]$, where $\mathbf{E}^Q[z_k] = \sum_s q_s \times z_{ks}$, for every security k and Q is called a *risk-neutral probability measure* (RNPM).

4. The Model with a Tax Authority

This section introduces taxation in the model discussed above and analyzes its implications. While section 4.1 discusses our main assumptions, section 4.2 examines the pricing effect for general policy designs.

[12]Note that the insiders' preference structure (no preferences for time-0 income) implies that an insider will only sell some fraction of the global bond to some other domestic investor in exchange for domestic securities. Moreover, insider's resources are (by assumption) completely determined by their initial portfolios. Thus, no domestic investor will be able to buy an additional position in the global bond from a foreign investor (since the latter are not allowed to hold domestic securities). In effect, the insider's aggregate stake in the international bond will remain unchanged after the trade took place.

4.1. Basic assumptions

Our analysis examines a flat capital income tax levied on interest income, dividends and capital gains. The tax code does not differentiate between domestic securities and the global bond. Accordingly, in the single-period model set-up our tax code coincides with a flat withholding tax on capital income defined as the sum of an investor's dividends, interests and capital gains. In our analysis tax revenues are equal to expenditure by the authority. Thus, we do not consider public goods but assume that tax revenues are immediately redistributed among domestic individuals as lump-sum payments. Note that differentiating between capital market participants and outsiders redistribution provides a second mechanism for the authority to alter equilibrium outcomes.[13]

The following assumptions 5 - 7 formally define our model set-up and introduce the corresponding notation.

Assumption 5. *The tax establishes an income tax on interests, dividends and capital gains. The tax function is linear with a tax rate $\tau > 0$ identical for all agents and all securities. In case of a negative tax base, the authority grants an immediate tax loss offset.*

Assumption 6. *After enacting the tax code, government chooses an expenditure policy offering lump-sum redistribution in the form of monetary transfer payments. The amount of redistribution offered to the cohort of insiders in time-1 is modeled by the random variable $L = (L_r, L_b)$. Insiders are well aware of the type of redistribution that the authority is going to apply. For the stand-in household L essentially represents a per capita transfer, which is internalized in its optimization problem as an additional source of income.*

Assumption 7. *The introduction of a tax authority does not alter beliefs and preferences of the stand-in household for time-1 income.*

Let $\mathcal{P}$ denote the policy design enacted by the authority. We characterize $\mathcal{P}$ by its tax rate τ and its amount of redistribution to insiders L. Moreover, let $B_s^{\mathcal{P}} = \sum_{i=1}^{K}(z_k - p_k^{\mathcal{P}}) + r_0 \times Y_0$ denote the stand-in household's tax base. The corresponding tax bill in state s sums up to $T_s^{\mathcal{P}} = \tau \times B_s^{\mathcal{P}}$, and $T_s^{\mathcal{P}} > 0$ ($T_s^{\mathcal{P}} < 0$) indicates that taxation reduces (increases) time-1 post-tax security income of the

[13]Moreover, note that a flat tax code provides a tax loss offset in case of a negative tax base. Accordingly, the aggregate tax revenue is negative in case of a negative aggregate tax base. Thus, a negative aggregate tax base results in negative redistribution, which corresponds to a per-capita or lump-sum tax for insiders.

stand-in household. However, the stand-in household internalizes the immediate redistribution L_s to insiders. Accordingly, with aggregate resources $X + (L - T^{\mathcal{P}})$ for the stand-in household, the $\mathcal{P}$-associated equilibrium security prices are given by

$$p_k^{\mathcal{P}} = \sum_{s \in \{r,b\}} \pi_s^{\mathcal{P}} \times z_{ks}^{\mathcal{P}} = \frac{1}{1 + r_0^{\mathcal{P}}} \times \mathbf{E}\left[\frac{u'(X + (L - T^{\mathcal{P}}))}{\mathbf{E}[u'(X + (L - T^{\mathcal{P}}))]} \times z_k^{\mathcal{P}}\right]. \qquad (3)$$

The first part of the equation elucidates the idea to disentangle the pricing effect of a particular policy design $\mathcal{P}$ into two sub-effects: (i) the *equilibrium effect* for the ESPs ($\pi \to \pi^{\mathcal{P}}$), and (ii) the *payoff effect* for the (post-tax) payoffs promised by the security to its holder ($z_k \to z_k^{\mathcal{P}}$). Moreover, the right hand side of equation (3) illustrates that the equilibrium effect may be separated into two sub-effects: (i.a) the effect on the risk-free post-tax interest rate ($r_0 \to r_0^{\mathcal{P}}$), and (i.b) the effect upon the stand-in household's marginal utilities and the corresponding risk-neutral probabilities ($q_0 \to q_0^{\mathcal{P}}$).

Obviously, all three effects may be interrelated in the current setting. This is due to the fact, that security prices determine the aggregate tax base which in turn determines the tax proceeds and thus redistribution possibilities within the economy. Given the exogenous aggregate pre-tax security income, the latter determines the aggregate well-being of insiders which eventually affects security prices by affecting marginal utilities of the stand-in household.

4.2. The pricing effect for general policy designs

Next, we discuss the pricing effect for general policy designs. Therefore, we call a security *procyclical* (*countercyclical*), if its pre-tax payoff is positively (negatively) correlated to the aggregate pre-tax payoff of the market portfolio. In our simple binomial model economy security k is procyclical (countercyclical), if (and only if) its time-1 pre-tax payoff in the recession state is smaller (larger) that its boom-state equivalent.[14]

The following proposition shows, that for a general policy design the pricing

[14] From an asset pricing perspective, a procyclical (countercyclical) security is characterized by a positive (negative) beta-coefficient (measured against the market portfolio). Moreover, as is well-known, as long as the stand-in household is risk-averse the expected post-tax excess return (*risk-premium*) is positive for a procyclical security. In contrast, for countercyclical securities the expected post-tax excess return is negative, since countercyclical securities provide a hedge against income risk.

effect depends on (a) the fact whether or not the policy design affects the RNPM of the economy and (b) the security's *cyclicality*.

Proposition 1 (Pricing effect for general policy designs)**.** *Suppose the government enforces a policy design $\mathcal{P}$. Then, the price of security k is given by*

$$p_k^{\mathcal{P}} = p_k + \frac{z_{kb} - z_{kr}}{1 + r_0} \times (q_b^{\mathcal{P}} - q_b), \tag{4}$$

where on the r.h.s. only the last term depends upon the prevailing policy design.

Proof: See appendix A Q.E.D.

There are four main findings from proposition 1. First, if the policy design does not affect the RNPM of the economy, then it does not induce a pricing effect, i.e. security prices do not reflect the level of the tax rate. Second, there is no pricing effect for a security with a risk-free pre-tax payoff.[15] Third, if the policy design affects the RNPM of the economy, then the pricing effect for the security is sensitive with respect to the level of the exogenous interest rate before taxes and the variability of the security's payoffs before taxes. Furthermore, in case of an equilibrium effect for the RNPM the proposition predicts a differentiating pricing effect, which will imply a substitution effect on the level of households' portfolios. Note that this finding is in sharp contrast to the small open economy finding in Sandmo (1989).

Finally, proposition 1 shows that the level of the prevailing tax rate affects the pricing effect only indirectly via $(q_b^{\mathcal{P}} - q_b)$, since all direct effects cancel each other out. This is due to the assumption of investors facing a global bond market. In this case, the interest rate r_0 (measured before taxes) is exogenous. In case of segmented bond markets optimizing households will force the interest rate to adjust as the (dynamic) analyses of Sialm (2005, 2006) (for a dividend and a consumption tax, respectively) and Rapp (2007) (for a comprehensive capital income tax) reveal. This effect for the pre-tax risk-free interest rate, which will only be observed in closed economies, induces a pricing effect that affects the price level of risk-free securities.

We close this section by discussing a stylized policy regime in which the au-

[15]Clearly, this is a direct implication of (a) the linear tax code, (b) markets that are in equilibrium, and (c) assumption 2: If the tax code offers immediate tax loss offset, financial markets do not offer arbitrage opportunities to their participants, and the risk-free global bond trades for an exogenous price, then any risk-free local stock must trade for a price that is independent of the tax rate.

thority immediately redistributes all tax proceeds to investors. We call such a policy regime *full-redistribution regime*, labeled $\mathcal{F}$. Technically, $\mathcal{F}$ is characterized by $L = T^{\mathcal{F}}$. Thus, redistribution (the sum of all per-capital transfers) exactly offsets aggregate tax payments and there is no effect upon marginal utilities of the stand-in household. While this implies that there is no effect for risk-neutral probabilities, the model still predicts an effect for ESPs of the economy. The latter effect is due to the exogenous risk-free pre-tax interest rate. However, given the observation of proposition 1 the first effect implies that there is no pricing effect under a full redistribution regime. In sum, linear taxation of capital income accompanied by a full-redistribution regime is a *neutral* policy design for our asset pricing model with a risk-averse stand-in household and an exogenous risk-free rate as it neither causes substitution nor income effects.

Again, note that the assumption of an exogenous risk-free interest rate is vitally important for the above result. If a closed economy with an endogenous risk-free interest rate is considered in contrast, then the equilibrium effect of increasing ESPs vanishes, since the pre-tax risk-free rate increases. Thus, in a closed economy with an endogenous risk-free interest rate the post-tax risk-free rate is independent of the tax rate in the case of a full-redistribution regime (e.g. Sialm, 2006; Rapp, 2007).

5. The pricing effect when tax proceeds are fully distributed to outsiders

This section studies the effect of a capital income tax under the assumption that tax proceeds are perfectly extracted from the cohort of insiders. The corresponding redistribution design is called no-redistribution regime and the associated policy design is labeled $\mathcal{N}$. Technically, it is characterized by $L = 0$.

In a first step, the sensitivity of the RNPM with respect to the tax rate τ is analyzed. Let $p_M^{\mathcal{N}}$ denote the price of the aggregated market portfolio given $\mathcal{N}$. Then, the corresponding state-s risk-neutral probability is characterized by

$$q_s^{\mathcal{N}} = \frac{u'((1-\tau) \times X_s + \tau p_M^{\mathcal{N}} + \tau \times Y_0)}{\mathbf{E}[u'((1-\tau) \times X + \tau p_M^{\mathcal{N}} + \tau \times Y_0)]} \times \phi_s. \tag{5}$$

Among others, the tax rate-sensitivity of $q_s^{\mathcal{N}}$ depends upon u, r_0, $\mathbf{E}[M]$ and $M_b - M_s$. Instead of assuming that preferences of the stand-in household satisfy certain conditions, our analysis presumes that the economy is *sufficiently volatile* in the sense of the following assumption 8.

Assumption 8. *For all tax rates, the aggregate tax base of the stand-in household is negative in the recession state. Formally,* $B_r^{\mathcal{N}} \leq 0$ *for all* $\tau \in [0, 1]$.

Assumption 8 presumes for the recession state that aggregate capital losses of the market portfolio are larger than corresponding dividends plus interest income from the international bond. As shown in appendix B this is equivalent to assuming that the aggregate time-1 pre-tax payoff of the market portfolio M is sufficiently volatile. Specifically, it is shown that the assumption is satisfied, if (a) the risk-free pre-tax interest rate is zero or (b) $(1 + (r_0/\phi_b)) \times X_r \leq X_b$, where $X = M + (1 + r_0) \times Y_0$.

With assumption 8 the tax-rate sensitivity of the numerator of equation (5) is zero or negative in the recession state. In case of the boom state, however, the corresponding sensitivity is positive. Since the sensitivity works in the opposite direction for the two states, the effect of the numerator dominates the effect in the denominator. The overall effect is summarized in the following corollary.

Corollary 1 (RNPM under no-redistribution)**.** *Suppose the government enforces a fixed policy design with a no-redistribution regime component. Then, the risk-neutral probability for the recession state decreases with the tax rate, whereas for the boom state it increases with the tax rate.*

Proof: See appendix C. Q.E.D.

The intuition of the corollary is that taxation reduces the variability of time-1 security income after taxes. Specifically, since the tax code provides full loss offset volatility reduces to zero for $\tau = 1$. In other words, time-1 income and corresponding marginal utilities become deterministic as τ approaches 1. Thus, $\lim_{\tau \to 1} q_s^{\mathcal{N}} = \phi_s$.

As a direct implication of the above corollary we find that a no-redistribution regime induces an equilibrium effect: The ESP for the boom state is increasing with the tax rate. To see this, note that $\pi_b^{\mathcal{N}} = \left(1 + r_0^{\mathcal{N}}\right)^{-1} \times q_b^{\mathcal{N}}$ and hence

$$\frac{\partial}{\partial \tau} \pi_b^{\mathcal{N}} = \frac{r_0}{(1 + r_0^{\mathcal{N}})^2} \times q_b^{\mathcal{N}} + \frac{1}{1 + r_0^{\mathcal{N}}} \times \frac{\partial}{\partial \tau} q_b^{\mathcal{N}} > 0.$$

The effect for the recession state ESP is ambiguous in general. However, for $r_0 = 0$

it is easy to see that the $\pi_r^{\mathcal{N}}$ decreases with the tax rate.

The following proposition derives the pricing effect of taxation in case of a no-redistribution regime. Using the results of proposition 1 it claims that (a) the pricing effect is sensitive with respect to the variability of a security's time-1 payoff before taxes and (b) the sign of the pricing effect is sensitive with respect to correlation between the security's pre-tax payoff and the pre-tax payoff of the market portfolio.

Proposition 2 (Pricing effect under no-redistribution)**.** *Suppose the government enforces a fixed policy design with a no-redistribution regime component. Then, the pricing effect for any security depends (affine) linearly upon the pre-tax variability of its time-1 pre-tax payoff. In particular, if the correlation of the security's payoff with the aggregate payoff of the market portfolio is positive, then its equilibrium post-tax price increases with the tax rate and vice versa.*

Proof: The claim is an immediate implication of proposition 1 and corollary 1. Specifically, the latter shows that under no-redistribution an increasing tax rate produces an increasing risk-neutral probability for the boom state, i.e. $q_b^{\mathcal{N}} - q_b > 0$ for all $\mathcal{N}$ with a strictly positive tax rate. Thus, if $z_{kb} - z_{kr}$ is positive (negative or zero), an increasing tax rate leads to an increasing (decreasing or stable) equilibrium price of security k. Q.E.D.

The above proposition claims that under no-redistribution the price of a procyclical (a counter-cyclical) security is positively (negatively) related to the tax rate in our model. Now, the market portfolio may be thought of as a procyclical security, since $M_b - M_r > 0$. Accordingly, in case of the no-redistribution regime, the current price of the market portfolio is positively correlated to the prevailing tax rate. Since an increasing price of the market portfolio implies lower expected before-tax returns in the future (and vice versa), our model predicts a negative tax rate sensitivity of the *ex-ante expected equity premium* (measured before taxes) in case of the no-redistribution regime. Moreover, our model then predicts that the *observed ex-post equity premium* (measured before taxes) is positively correlated to the tax rate, since an increasing price of the market portfolio implies higher realized returns before taxes (and vice versa).

Although our analysis is concerned with a pure exchange economy, proposition 2 sheds light on what we may expect in a classical production economy: Due to the fact that the price of a procyclical (a counter-cyclical) security is predicted to be positively (negatively) related to the tax rate, the cost of capital for corresponding investment projects is negatively (positively) related to the tax rate. This means that in an economy with endogenous production an increasing tax rate is supposed to generally affect investment decision of firms by favoring cyclical investment projects.

6. Conclusion

We examine the effect of a linear capital income tax upon security prices in a single-period pure exchange economy with binomial uncertainty and an exogenous interest rate. As such, the model captures features of an economy with perfectly integrated bond markets but locally segmented stock markets. Our analysis shows that the pricing effect, i.e. the effect of taxation upon security prices is a function in (a) the covariance between the pre-tax payoffs of the security and the aggregated market portfolio, (b) the exogenous pre-tax risk-free rate and most important (c) the tax effect for the risk-neutral probabilities of the domestic stock market. Thereby, the latter turns out to be sensitive to the redistribution regime enacted by the authority.

We illustrate that if the authority redistributes tax proceeds within the cohort of market participants, then marginal utilities of the representing household are unaffected by taxation and prices of securities do not reflect the level of the prevailing tax rate. However, if in contrast taxation is used as a policy tool to transfer consumption possibilities to non-market participants, then taxation may considerably affect economic outcomes. In this case marginal utilities of the stand-in household are affected by taxation and the model predicts a differentiating pricing effect. Specifically, while the price of procyclical securities increases with an increasing tax rate, the price of counter-cyclical securities reacts in the opposite way. In effect, the model predicts a substitution effect on the level of household portfolios that may affect investment decisions of firms.

In sum, we note that our analysis reveals that the effects of taxation are highly sensitive with respect to the corresponding redistribution regime. Now, there seem to be two arguments in favor of the no-redistribution regime. First, it is not clear at all whether individuals really account for government transfers in their portfolio choice decisions. Second, there is empirical evidence for limited market participation as pioneered by Mankiw and Zeldes (1991) and it seems fair to presume that redistribution does not solely go to privileged market participants but specifically to relatively poor non-market participants. However, the question of which assumption is more appropriate remains an empirical one and more research should be devoted to these issues.

Appendix

A. Proof of proposition 1

This appendix proves proposition 1. We start with an observation, which will be used in the subsequent proof of the proposition.

Observation: For any policy design $\mathcal{P}$ we have $q_r^{\mathcal{P}} + q_b^{\mathcal{P}} = q_r + q_b = 1$, which implies $q_b^{\mathcal{P}} - q_b = -(q_r^{\mathcal{P}} - q_r)$. Thus, for any pair (a_r, a_b) we may write

$$\sum_{s\in\{r,b\}} q_s^{\mathcal{P}} \times a_s = \left[\sum_{s\in\{r,b\}} q_s \times a_s\right] + (a_b - a_r) \times (q_b^{\mathcal{P}} - q_b). \tag{A.1}$$

Proof of proposition 1: Recall that $z_{ks}^{\mathcal{P}} = (1-\tau) \times z_{ks} + \tau \times p_k^{\mathcal{P}}$. Thus,

$$\begin{aligned} p_k^{\mathcal{P}} &= \frac{1}{1+r_0^{\mathcal{P}}} \times \sum_{s\in\{r,b\}} q_s^{\mathcal{P}} \times z_k^{\mathcal{P}} \\ &= \left[\frac{1}{1+r_0^{\mathcal{P}}} \times \sum_{s\in\{r,b\}} q_s^{\mathcal{P}} \times (1-\tau) \times z_{ks}\right] + \frac{\tau}{1+r_0^{\mathcal{P}}} \times p_k^{\mathcal{P}}. \end{aligned} \tag{A.2}$$

Subtracting $(\tau \times p_k^{\mathcal{P}})/(1+r_0^{\mathcal{P}})$ from both sides and multiplying with $(1+r_0^{\mathcal{P}})/(1+r_0^{\mathcal{P}}-\tau)$ then gives

$$p_k^{\mathcal{P}} = \frac{1}{1+r_0} \times \sum_{s\in\{r,b\}} q_s^{\mathcal{P}} \times z_{ks}, \tag{A.3}$$

since $(1+r_0^{\mathcal{P}}-\tau) = (1-\tau) \times (1+r_0)$. The final step is now to apply the above observation (A.1) in order to re-arrange equation (A.3) to

$$p_k^{\mathcal{P}} = \underbrace{\left[\frac{1}{1+r_0} \times \sum_{s\in\{r,b\}} q_s \times z_{ks}\right]}_{=p_k} + \frac{(z_{kb} - z_{kr})}{1+r_0} \times (q_b^{\mathcal{P}} - q_b), \tag{A.4}$$

which then proves the proposition.

B. Discussion of assumption 8

To gain deeper insight into assumption 8 note that equation (A.4) holds for any policy design $\mathcal{P}$. Hence, the price of the aggregate market portfolio $p_M^{\mathcal{P}}$ is given by

$$p_M^{\mathcal{P}} = \sum_{k=1}^{K} p_k^{\mathcal{P}} = \sum_{s\in\{r,b\}} \frac{q_s^{\mathcal{P}}}{1+r_0} \times M_s.$$

Accordingly, in the recession state the aggregate tax base is given by

$$\begin{aligned} B_r^{\mathcal{P}} &= \left(M_r - \sum_{k=1}^{K} p_k^{\mathcal{P}} \right) + r_0 \times Y_0 \\ &= (M_r + (1+r_0) \times Y_0) - \sum_{s \in \{r,b\}} \frac{q_s^{\mathcal{P}}}{1+r_0} \times (M_s + (1+r_0) \times Y_0). \end{aligned}$$

Thus, with $X_s = M_s + (1 + r_0) \times Y_0$ the aggregate tax base is negative, if (and only if)

$$X_r \leq \frac{q_b^{\mathcal{P}}}{q_b^{\mathcal{P}} + r_0} \times X_b \quad \Leftrightarrow \quad \left(1 + \frac{r_0}{q_b^{\mathcal{P}}}\right) \times X_r \leq X_b. \tag{B.1}$$

Moreover, due to the assumption of a risk-averse stand-in household and assumption 3 claiming $M_r < M_b$, we have $q_b^{\mathcal{N}} < \phi_b$. With $r_0 \geq 0$ this implies $(1 + r_0/\phi_b) \times X_r \leq (1 + r_0/q_b^{\mathcal{N}}) \times X_r$. Thus, $(1 + r_0/\phi_b) \times X_r \leq X_b$ or equivalently $X_r \leq (1 + r_0/\phi_b)^{-1} \times X_b$ turns out to be a sufficient condition for (B.1).

C. Proof of Corollary 1

This appendix proves corollary 1, where the government is supposed to implement a no-redistribution regime. Therefore, let $\mathcal{N}_1$ and $\mathcal{N}_2$ denote policy designs with no-redistribution expenditure component and associated tax rates that satisfy $\tau_1 < \tau_2$. Moreover, recall $X = M + (1 + r_0) \times Y_0$. Then

$$\begin{aligned} u'(X_r + T_r^{\mathcal{N}_2}) &\leq u'(X_r + T_r^{\mathcal{N}_1}) \\ u'(X_b + T_b^{\mathcal{N}_2}) &> u'(X_b + T_b^{\mathcal{N}_1}), \end{aligned}$$

since by assumption 8 we have $T_r^{\mathcal{N}_2} \geq T_r^{\mathcal{N}_1} \geq 0$ and $T_b^{\mathcal{N}_2} \leq T_b^{\mathcal{N}_1} \leq 0$. Defining a_r and a_b by

$$\begin{aligned} a_r &= \frac{u'(X_r + T_r^{\mathcal{N}_2})}{u'(X_r + T_r^{\mathcal{N}_1})} \leq 1 \\ a_b &= \frac{u'(X_b + T_b^{\mathcal{N}_2})}{u'(X_b + T_b^{\mathcal{N}_1})} > 1 \end{aligned}$$

we may write

$$\mathbf{E}[u'(X + T^{\mathcal{N}_2})] = a_r \times \sum_{s \in \{r,b\}} \phi_s \times \frac{a_s}{a_r} \times u'(X_s + T_s^{\mathcal{N}_1})$$

and

$$\mathbf{E}[u'(X + T^{\mathcal{N}_2})] = a_b \times \sum_{s \in \{r,b\}} \phi_s \times \frac{a_s}{a_b} \times u'(X_s + T_s^{\mathcal{N}_1})$$

In particular, this implies

$$a_r \times \mathbf{E}[u'(X + T^{\mathcal{N}_1})] < \mathbf{E}[u'(X + T^{\mathcal{N}_2})] < a_b \times \mathbf{E}[u'(X + T^{\mathcal{N}_1})]$$

since $a_r \leq 1, a_b > 1$ and $\phi(r)$ as well as $\phi(b)$ are greater zero (assumption 1). Now, note that $a_r \times \mathbf{E}[u'(X + T^{\mathcal{N}_1})] < \mathbf{E}[u'(X + T^{\mathcal{N}_2})]$ is equivalent to

$$\frac{a_r}{\mathbf{E}[u'(X + T^{\mathcal{N}_2})]} < \frac{1}{\mathbf{E}[u'(X + T^{\mathcal{N}_1})]}.$$

Multiplying the last inequality with $u'(X_r + T_r^{\mathcal{N}_1})$ yields

$$\frac{u'(X_r + T_r^{\mathcal{N}_2})}{\mathbf{E}[u'(X + T^{\mathcal{N}_2})]} \leq \frac{u'(X_r + T_r^{\mathcal{N}_1})}{\mathbf{E}[u'(X + T^{\mathcal{N}_1})]}.$$

The latter, however, implies $q_r^{\mathcal{N}_2} \leq q_r^{\mathcal{N}_1}$. Going a similar way yields

$$\frac{u'(X_b + T_b^{\mathcal{N}_2})}{\mathbf{E}[u'(X + T^{\mathcal{N}_2})]} \geq \frac{u'(X_b + T_b^{\mathcal{N}_1})}{\mathbf{E}[u'(X + T^{\mathcal{N}_1})]}$$

and, thus, $q_b^{\mathcal{N}_2} \geq q_b^{\mathcal{N}_1}$.

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